M. Math 1st/B. Math 3rd 2009-2010 Topology Midterm examination

points: 8x5=40 18-09-2009 Time: 3hrs

Show complete work. You may assume any results proved in the class, but need to quote them correctly. Each question is worth 5 points.

1) Let X be a topological space. Formulate and prove the statement 'subnet of a net is a net'. Show that if a net converges to a point, subnet of the net converges to the same point.

2) Let X be an infinite set with a regular topology τ on it. Show that there exists a sequence $\{x_n\}_{n\geq 1}$ and a sequence of pair-wise disjoint open sets $\{U_n\}_{n\geq 1}$ such that $x_n \in U_n$.

3) Let X be a topological space. Give the complete details to show that the set of real-valued continuous functions on X is a vector space.

4) Let $\{X_\alpha\}$ be a family of T_2 topological spaces. Let $X = \prod X_\alpha$. For a fixed α, show that the canonical embedding of X_α into X is a homeomorphism onto a closed subset of X.

5) Let X be a topological space. Show that a bijection $f : X \to X$ is a homeomorphism if and only if $f(\overline{A}) = \overline{f(A)}$

6) Give with details, examples of topological spaces X, Y such that X is homeomorphic to a subset of Y, Y is homeomorphic to a subset of X, but X and Y are not homeomorphic.

7) Show that an open set in R^n has at most countably many components.

8) Let τ be the usual topology on the set of real numbers R. Let τ' be the topology with all sets of the form (b, ∞) as a subbasis. Show that for any topological space X, $f : X \to (R, \tau)$ is lower semicontinuous if and only if $f : X \to (R, \tau')$ is continuous.